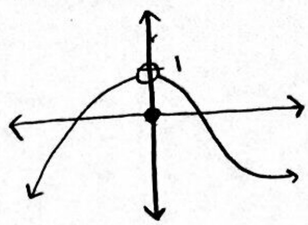
Quiz 4

September 9, 2016

Show all work and circle your final answer.

1. Sketch a graph of a function f satisfying

$$\begin{cases} \lim_{x\to 0^-} f(x) = 1 \\ \lim_{x\to 0^+} f(x) = 1 \\ f(0) = 0 \end{cases}$$



   (a) What is $\lim_{x\to 0} f(x)$? $\boxed{1}$

   (b) Is f continuous at $x = 0$? Why?

   No, because $\lim_{x\to 0} f(x) \neq f(0)$.

2. Consider $f(x) = \begin{cases} x^2 - \sin(\pi x) - 1 & x < 1 \\ (x-1)^2 & x \geq 1 \end{cases}$.

   At what values (if any) is $f(x)$ discontinuous? Why?

   $f(x)$ is continuous at all $x \neq 1$ since $\sin(\pi x)$ and polynomials are continuous. We check at $x=1$:

   $$\left.\begin{aligned} \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} x^2 - \sin(\pi x) - 1 = 1^2 - \sin\pi - 1 = 0 \\ \lim_{x\to 1^+} f(x) &= \lim_{x\to 1} (x-1)^2 = 0 = f(1) \end{aligned}\right\} \text{continuous at } x=1$$

3. Let $f(x) = \begin{cases} x^2 + cx & x < 1 \\ cx^4 + 2x & x \geq 1 \end{cases}$.

   For what value(s) of c, if any, is f continuous everywhere?

   $f(x)$ is continuous at all $x \neq 1$. We check at $x=1$:

   $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 + cx = 1 + c$

   $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} cx^4 + 2x = c + 2 = f(1)$

   We want $1 + c = c + 2$

   $\quad\quad\quad\; -c \quad -c$

   $1 = 2$ ← never true, so f is never continuous at $x=1$.